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# Fixed Point Approximation of Mappings Satisfying the $(RCSC)$-Condition in Hyperbolic Spaces

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**Abstract.** In this paper, we investigate fixed point approximation of mappings satisfying the Reich–Chatterjea–Suzuki-type condition ((RCSC)-condition) in uniformly convex hyperbolic spaces. By using the $K$-iteration process, we establish $\Delta$-convergence and strong convergence results under suitable assumptions. An example illustrating the applicability of the obtained results is presented. Furthermore, an application to a nonlinear Fredholm integral equation is provided. The obtained results extend and complement several existing convergence theorems for generalized nonexpansive mappings.

## 1. Introduction

Fixed point theory is one of the central areas of nonlinear analysis and has numerous applications in optimization, differential equations, economics, engineering, and other branches of applied mathematics. The celebrated Banach contraction principle [1] initiated the development of modern fixed point theory and inspired a large number of extensions and generalizations. Among the most notable contributions are those due to Kannan [2], Chatterjea [3], Reich [4], Hardy–Rogers [5], and Ćirić [6], which significantly broadened the class of mappings for which fixed point results can be established.

Alongside the study of fixed point existence, the approximation of fixed points has attracted considerable attention. Since exact fixed points are often difficult to determine, numerous iterative methods have been developed to approximate them. In particular, convergence properties of iterative schemes have been extensively investigated for nonexpansive and generalized nonexpansive mappings in Banach spaces and, more recently, in nonlinear geometric settings. In this direction, important contributions were made by Takahashi [7], Goebel and Kirk [8], and Kohlenbach [9]. Furthermore, Kirk and Panyanak [10] introduced the notion of $\Delta$-convergence in $CAT(0)$ spaces, which has become an important tool in the study of iterative approximation methods in geodesic and hyperbolic spaces. Since then, a variety of convergence results have been obtained in CAT(0) spaces, hyperbolic spaces, and related nonlinear structures [11]-[14].

In 2008, Suzuki [15] introduced Condition-(C), which defines a class of mappings lying between nonexpansive and quasi-nonexpansive mappings. This condition has attracted considerable attention and led to several further generalizations. In particular, Karapınar [16] introduced the $(RSC)$, $(RCSC)$ and $(HRSC)$-conditions as generalized Suzuki-type conditions and established fundamental properties of the

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corresponding mapping classes. Subsequently, various convergence and approximation results for generalized Suzuki-type mappings have been obtained in CAT(0), geodesic, and hyperbolic spaces [17]-[19].

Several multi-step iteration processes have recently been introduced to improve the efficiency of fixed point approximation methods. Let $\Omega$ be a nonempty convex subset of a Banach space $\tilde{N}$ and let $\Upsilon : \Omega \longrightarrow \Omega$ be a mapping,where $\{\alpha_k\}$ and $\{\beta_k\}$ are sequences in $[0, 1]$ for $k \geq 0$.

Ullah and Arshad [20] introduced the $M$-iteration process as follows:

$$\begin{cases} \varrho_1 \in \Omega \\ \varrho_{k+1} = \Upsilon \ell_k \\ \ell_k = \Upsilon \psi_k \\ \psi_k = (1-\alpha_k)\,\varrho_k + \alpha_k \Upsilon \varrho_k \,. \end{cases}$$

Ullah and Arshad [21] also introduced another three-step iteration process known as the $M^*$-iteration process, defined as:

$$\begin{cases} \varrho_1 \in \Omega \\ \varrho_{k+1} = \Upsilon \ell_k \\ \ell_k = \Upsilon\left((1-\alpha_k)\,\varrho_k + \alpha_k \Upsilon \psi_k\right) \\ \psi_k = (1-\beta_k)\,\varrho_k + \beta_k \Upsilon \varrho_k \,. \end{cases}$$

In 2012, Hussain et al. [22] introduced a three step iterative process called the $K$-iteration process defined by

$$\begin{cases} \varrho_1 \in \Omega \\ \varrho_{k+1} = \Upsilon \ell_k \\ \ell_k = \Upsilon\left((1-\alpha_k)\,\Upsilon\varrho_k + \alpha_k \Upsilon \psi_k\right) \\ \psi_k = (1-\beta_k)\,\varrho_k + \beta_k \Upsilon \varrho_k \,. \end{cases} \tag{1}$$

In [22], it has been demonstrated the efficiency of the $K$-iteration process through numerical comparisons and established weak and strong convergence results for Suzuki generalized nonexpansive mappings in uniformly convex Banach spaces.

Recently, Saleem et al. [18] studied fixed point approximation of mappings satisfying the $(RCSC)$-condition in $CAT(0)$ by employing the $M$-iteration process. They established convergence results for this class of generalized Suzuki-type mappings.

Motivated by the above developments, in this paper we study fixed point approximation of mappings satisfying the $(RCSC)$-condition by means of the $K$-iteration process in complete uniformly convex hyperbolic spaces. We establish $\Delta$-convergence and strong convergence results for the proposed iterative scheme and provide an illustrative example together with an application to a nonlinear Fredholm integral equation. Furthermore, a numerical comparison with the $M$-iteration and $M*$-iteration processes is presented. Our results complement and extend existing convergence results in the literature.

## 2. Preliminaries

Let $\Omega \neq \emptyset \subseteq \tilde{N}$ where $(\tilde{N}, d_w)$ is a metric space and $\Upsilon : \Omega \longrightarrow \Omega$ is a self mapping, then a point $\eth \in \Omega$ is a fixed point of the mapping $\Upsilon$ if $\Upsilon\eth = \eth$. The set $F(\Upsilon)$, i.e., $F(\Upsilon) = \{\eth \in \Omega : \Upsilon\eth = \eth\}$ denotes the set of all fixed points of mapping $\Upsilon$.

**Definition 2.1.** 1. *The mapping $\Upsilon$ is called nonexpansive mapping if for $\varrho, \ell \in \Omega$, we have*

$$d_w(\Upsilon\varrho, \Upsilon\ell) \leq d_w(\varrho, \ell)\,.$$

2. *A self-mapping $\Upsilon : \Omega \to \Omega$ is said to be quasi-nonexpansive mapping if the set of fixed point $F(\Upsilon) \neq \emptyset$ and for all $\varrho \in \Omega$ and $\eth \in F(\Upsilon)$, we have*

$$d_w(\Upsilon\varrho, \eth) \leq d_w(\varrho, \eth)\,.$$

3. *([15]) Let $\Omega \neq \emptyset \subseteq \tilde{N}$ where $(\tilde{N}, d_w)$ is a metric space. Then $\Upsilon : \Omega \to \Omega$ satisfies Condition-(C) if we have*

$$\frac{1}{2} d_w(\varrho, \Upsilon\varrho) \leq d_w(\varrho, \ell) \Rightarrow d_w(\Upsilon\varrho, \Upsilon\ell) \leq d_w(\varrho, \ell)$$

*for each $\varrho, \ell \in \Omega$.*

4. *([16]) Let $\Omega \neq \emptyset \subseteq \tilde{N}$ (metric space). Then $\Upsilon : \Omega \to \Omega$ satisfies Reich-Chatterjea-Suzuki-(C) condition ((RCSC)-condition) if*

$$\begin{aligned} \frac{1}{2} d_w(\varrho, \Upsilon\varrho) &\leq d_w(\varrho, \ell) \text{ implies that} \\ d_w(\Upsilon\varrho, \Upsilon\ell) &\leq \frac{1}{3}\left[d_w(\varrho, \ell) + d_w(\Upsilon\varrho, \ell) + d_w(\varrho, \Upsilon\ell)\right] \end{aligned}$$

*for $\varrho, \ell \in \Omega$.*

**Example 2.2.** *Let $\Omega = [0, 3]$ with the usual metric $d_w(\varrho, \ell) = |\varrho - \ell|$ and define $\Upsilon : \Omega \to \Omega$ by*

$$\Upsilon(\varrho) = \begin{cases} 0 & \text{if } \varrho \in [0, 3), \\ \frac{4}{3} & \text{if } \varrho = 3. \end{cases}$$

*Then $F(\Upsilon) = \{0\}$. We show that $\Upsilon$ satisfies the (RCSC)-condition.*

**Case 1.** *Let $\varrho, \ell \in [0, 3)$. Then $\Upsilon(\varrho) = \Upsilon(\ell) = 0$ and hence the required inequality is immediate.*

**Case 2.** *Let $\varrho = 3$ and $\ell \in [0, 3)$. If*

$$\frac{1}{2} d_w(3, \Upsilon(3)) \leq d_w(3, \ell),$$

*then $\frac{5}{6} \leq 3 - \ell$, so $\ell \leq \frac{13}{6}$. Therefore,*

$$d_w(\Upsilon(3), \Upsilon(\ell)) = \frac{4}{3}$$

*and*

$$\frac{1}{3}\left[d_w(3, \ell) + d_w(\Upsilon(3), \ell) + d_w(3, \Upsilon(\ell))\right] = \frac{1}{3}\left[(3 - \ell) + \left|\frac{4}{3} - \ell\right| + 3\right] \geq \frac{4}{3}.$$

*Thus the (RCSC)-condition holds.*

**Case 3.** *Let $\varrho \in [0, 3)$ and $\ell = 3$. If*

$$\frac{1}{2} d_w(\varrho, \Upsilon(\varrho)) \leq d_w(\varrho, 3),$$

*then $\frac{\varrho}{2} \leq 3 - \varrho$, so $\varrho \leq 2$. Hence,*

$$d_w(\Upsilon(\varrho), \Upsilon(3)) = \frac{4}{3}$$

*and*

$$\frac{1}{3}\left[d_w(\varrho, 3) + d_w(\Upsilon(\varrho), 3) + d_w(\varrho, \Upsilon(3))\right] = \frac{1}{3}\left[(3 - \varrho) + 3 + \left|\varrho - \frac{4}{3}\right|\right] \geq \frac{4}{3}.$$

*Therefore $\Upsilon$ satisfies the (RCSC)-condition.*

*Moreover, $\Upsilon$ does not satisfy the Condition-(C). Indeed, for $\varrho = 3$ and $\ell = 2$, we have*

$$\frac{1}{2} d_w(3, \Upsilon(3)) = \frac{5}{6} \leq d_w(3, 2) = 1,$$

*but*

$$d_w(\Upsilon(3), \Upsilon(2)) = \frac{4}{3} > 1 = d_w(3, 2).$$

*Hence $\Upsilon$ satisfies the (RCSC)-condition but does not satisfy Condition-(C).*

**Remark 2.3.** *The previous example shows that there exist mappings satisfying the* (*RCSC*)*-condition which do not satisfy Condition-*(*C*)*. Hence, the* (*RCSC*)*-condition is a proper generalization of Condition-*(*C*)*.*

Let $(\tilde{N}, d_w)$ be a metric space and let $[0,l] \subset \mathbb{R}$. For two points $\varrho, \ell \in \tilde{N}$, a mapping $\xi : [0,1] \to \tilde{N}$ is called a path joining $\varrho$ and $\zeta$ whenever $\xi(0) = \varrho$ and $\xi(1) = \ell$. The path $\xi$ is said to be a geodesic if

$$d_w(\xi(s), \xi(t)) = d_w(\xi(0), \xi(1))|s-t|, \text{ for every } s, t \in [0,1].$$

A metric space $(\tilde{N}, d_w)$ is called a geodesic space if every pair of points in $\tilde{N}$ can be joined by a geodesic path. In general, the geodesic segment joining two points need not be unique.

The following precise formulation of hyperbolic spaces was introduced by Kohlenbach [10].

**Definition 2.4.** *([9]) Let $(\tilde{N}, d_w)$ be a metric space and the let $W : \tilde{N} \times \tilde{N} \times [0,1] \to \tilde{N}$ be a mapping. Then the triplet $(\tilde{N}, d_w, W)$ is called a W-hyperbolic space if the following conditions hold for all $\varrho, \ell, \psi, \omega \in \tilde{N}$ and $\mu, \theta \in [0,1]$ :*

(W1) $d_w(\psi, W(\varrho, \ell, \mu)) \leq (1-\mu) d_w(\psi, \varrho) + \mu d_w(\psi, \ell)$;
(W2) $d_w(W(\varrho, \ell, \mu), W(\varrho, \ell, \theta)) = |\mu - \theta| d_w(\varrho, \ell)$;
(W3) $W(\varrho, \ell, \mu) = W(\ell, \varrho, 1-\mu)$;
(W4) $d_w(W(\varrho, \psi, \mu), W(\ell, \omega, \mu)) \leq (1-\mu) d_w(\varrho, \ell) + \mu d_w(\psi, \omega)$.

For simplicity, we use the notation

$$W(\varrho, \ell, \mu) = (1-\mu)\varrho \oplus \mu \ell$$

for a point in a $W$-hyperbolic space. For $\varrho, \ell \in \tilde{N}$, the geodesic segment joining $\varrho$ and $\ell$ is denoted by

$$[\varrho, \zeta] = \{(1-\mu)\varrho \oplus \mu\zeta : \mu \in [0,1]\}.$$

A subset $\Omega$ of $W$-hyperbolic space $(\tilde{N}, d_w, W)$ is called convex whenever $[\varrho, \ell] \subset \Omega$ for every $\varrho, \ell \in \Omega$.

**Definition 2.5.** *([13]) A W-hperbolic space $(\tilde{N}, d_w, W)$ is called uniformly convex (UCW−hyperbolic space) if for every $\varepsilon \in (0,2]$ and $t > 0$, there exists $\delta \in (0,1]$ such that whenever $\varrho, \ell, \psi \in \tilde{N}$ satisfy $d_w(\varrho, \psi) \leq t$, $d_w(\ell, \psi) \leq t$ and $d_w(\varrho, \ell) \geq \varepsilon t$, then*

$$d_w\left(\frac{1}{2}\varrho \oplus \frac{1}{2}\ell, \psi\right) \leq (1-\delta)t.$$

Let $\{\varrho_k\}$ be a bounded sequence in a hyperbolic space $(\tilde{N}, d_w, W)$ and $\Omega$ a nonempty subset of $\tilde{N}$. A functional $r(., \{\varrho_k\}) : \tilde{N} \to [0, +\infty)$ can be defined as follows:

$$r(\ell, \{\varrho_k\}) = \limsup_{k \to +\infty} d_w(\ell, \varrho_k).$$

The asymptotic radius of $\{\varrho_k\}$ with respect to $\Omega$ is described as

$$r(\Omega, \{\varrho_k\}) = \inf\{r(\ell, \{\varrho_k\}) : \ell \in \Omega\}.$$

A point $\varrho$ in $\Omega$ is called as an asymptotic center of $\{\varrho_k\}$ with respect to $\Omega$ if

$$r(\varrho, \{\varrho_k\}) = r(\Omega, \{\varrho_k\}).$$

$A(\Omega, \{\varrho_k\})$ is denoted as the set of all asymptotic centers of $\{\varrho_k\}$ with respect to $\Omega$.

**Definition 2.6.** *([23]) Let $\{\varrho_k\}$ be a bounded sequence in a W-hyperbolic space $(\tilde{N}, d_w, W)$. The sequence $\{\varrho_k\}$ is said to $\Delta$-converge to a point $\varrho \in \tilde{N}$ if $\varrho$ is the unique asymptotic center of every subsequence of $\{\varrho_k\}$.*

Let $(\tilde{N}, d_w)$ be a $W$-hyperbolic space and let $\Omega \subset \tilde{N}$ be nonempty. A sequence $\{\varrho_k\}$ in $\tilde{N}$ is called Fejér monotone with respect to $\Omega$ if

$$d_w(\eth, \varrho_{k+1}) \leq d_w(\eth, \varrho_k)$$

for every $\eth \in \Omega$ and for all $k \geq 0$.

**Definition 2.7.** *([24]) Let $\Upsilon : \Omega \to \Omega$ be a mapping with $F(\Upsilon) \neq \emptyset$. We say that $\Upsilon$ satisfies Condition $(I)$ if there exists a function $f : [0, +\infty) \to [0, +\infty)$ such that:*

1. *$f(r) > 0$ for all $r > 0$ and $f(0) = 0$.*
2. *$d_w(\varrho, \Upsilon(\varrho)) \geq f(d_w(\varrho, F(\Upsilon)))$ for every $\varrho \in \Omega$, where*

$$d_w(\varrho, F(\Upsilon)) = \inf\{d_w(\varrho, \ell) : \ell \in F(\Upsilon)\}.$$

**Proposition 2.8.** *([13]) Let $(\tilde{N}, d_w, W)$ be a complete UCW-hyperbolic space and let $\Omega$ be a nonempty closed convex subset of $\tilde{N}$. Then every bounded sequence in $\tilde{N}$ admits a unique asymptotic center with respect to $\Omega$.*

**Lemma 2.9.** *[16] Let $\Omega$ be a nonempty closed and convex subset of W-hyperbolic space $(\tilde{N}, d_w, W)$. If $\Upsilon : \Omega \to \Omega$ satisfies the $(RCSC)$-condition, then*

$$d_w(\varrho, \Upsilon\ell) \leq 9 d_w(\Upsilon\varrho, \varrho) + d_w(\varrho, \ell)$$

*for all $\varrho, \ell \in \Omega$*

**Remark 2.10.** *Substituting $\ell = \eth$ with $\eth \in F(\Upsilon)$ into the $(RCSC)$-condition and using $\Upsilon\eth = \eth$ yields*

$$d_w(\Upsilon\varrho, \eth) \leq \frac{1}{3}\left[2d_w(\varrho, \eth) + d_w(\Upsilon\varrho, \eth)\right].$$

*Rearranging gives*

$$3d_w(\Upsilon\varrho, \eth) \leq 2d_w(\varrho, \eth) + d_w(\Upsilon\varrho, \eth).$$

*and therefore*

$$d_w(\Upsilon\varrho, \eth) \leq d_w(\varrho, \eth)$$

*for all $\varrho \in \Omega$ and $\eth \in F(\Upsilon)$. Hence $\Upsilon$ is quasi- nonexpansive.*

**Lemma 2.11.** *([13]) Let $\{\varrho_k\}$ be a bounded sequence in a W-hyperbolic space $(\tilde{N}, d_w, W)$ and assume $A(\Omega, \{\varrho_k\}) = \{\psi\}$. Let $\{r_k\}$ and $\{s_k\}$ be two real sequences such that $r_k \geq 0$ for all $k \in \mathbb{N}$, $\limsup_{k\to\infty} r_k \leq 1$ and $\limsup_{k\to\infty} s_k \leq 0$. Suppose that $\ell \in \Omega$ and there exist $m, k_0 \in \mathbb{N}$ such that*

$$d_w(\ell, \varrho_{k+m}) \leq r_k d_w(\psi, \varrho_k) + s_k$$

*for every $k \geq k_0$. Then, $\ell = \psi$.*

**Lemma 2.12.** *([25]) Let $\{\varrho_k\}$ be a sequence in a W-hyperbolic space $(\tilde{N}, d_w, W)$ and let $\Omega \subset \tilde{N}$ be nonempty. Assume that $\{\varrho_k\}$ is Fejér monotone with respect to $\Omega$. If $A(\Omega, \{\varrho_k\}) = \{\varrho\}$ and $A\left(\tilde{N}, \left\{\varrho_{k_j}\right\}\right) \subseteq \Omega$ for every subsequence $\left\{\varrho_{k_j}\right\}$ of $\{\varrho_k\}$, then the sequence $\{\varrho_k\}$ $\Delta$-converges to $\varrho \in \Omega$.*

**Lemma 2.13.** *([26]) Let $(\tilde{N}, d_w, W)$ be a complete UCW-hyperbolic space and let $\omega \in \tilde{N}$. Assume that $\{\alpha_k\} \subset (0, 1)$ satisfies $0 < \alpha \leq \alpha_k \leq \beta < 1$ for all $k \in \mathbb{N}$. Let $\{\varrho_k\}$ and $\{\zeta_k\}$ be sequences in $\tilde{N}$ such that, for some $r \geq 0$,*

$$\limsup_{k\to+\infty} d_w(\varrho_k, \omega) \leq r, \limsup_{k\to+\infty} d_w(\ell_k, \omega) \leq r \text{ and } \lim_{k\to+\infty} d_w(\alpha_k \ell_k \oplus (1-\alpha_k)\varrho_k, \omega) = r.$$

*Then, $\lim_{k\to+\infty} d_w(\ell_k, \varrho_k) = 0$.*

## 3. Main Results

In the section, we extend the $K$-iteration process given by (1) to $W$-hyperbolic spaces. Let $(\tilde{N}, d_w, W)$ be a $W$-hyperbolic space and let $\Omega$ be a nonempty convex subset of $\tilde{N}$. For a mapping $\Upsilon : \Omega \to \Omega$, we consider the following modified $K$-iteration process:

$$\begin{cases} \varrho_1 \in \Omega \\ \varrho_{k+1} = \Upsilon \ell_k \\ \ell_k = \Upsilon\left((1-\alpha_k)\Upsilon \varrho_k \oplus \alpha_k \Upsilon \psi_k\right) \\ \psi_k = (1-\beta_k)\varrho_k \oplus \beta_k \Upsilon \varrho_k , \end{cases} \tag{2}$$

where $\{\alpha_k\}$ and $\{\beta_k\}$ are sequences in $[0,1]$ for $k \geq 0$.

**Lemma 3.1.** *Let $(\tilde{N}, d_w, W)$ be a complete UCW-hyperbolic space and let $\Omega$ be a nonempty closed convex subset of $\tilde{N}$. Assume that $\Upsilon : \Omega \to \Omega$ satisfies (RCSC)-condition and that $F(\Upsilon) \neq \emptyset$. Let $\{\varrho_k\}$ be the sequence generated by the iteration process (2), where $\alpha_k, \beta_k \in [\alpha, \beta]$ for some $\alpha, \beta \in (0,1)$. Then, the following results hold:*

(i) $\lim_{k\to+\infty} d_w(\varrho_k, ð)$ exists for all $ð \in F(\Upsilon)$.
(ii) $\lim_{k\to+\infty} d_w(\varrho_k, \Upsilon(\varrho_k)) = 0$.

*Proof.* (i) Let $ð \in F(\Upsilon)$. By the convexity of the metric combination and applying the induction-bound from the Remark 2.10 (so that $d_w(\Upsilon^k \varrho, ð) \leq d_w(\Upsilon \varrho, ð) \leq d_w(\varrho, ð)$ for all $k \geq 1$), we obtain

$$\begin{aligned} d_w(\psi_k, ð) &= d_w((1-\beta_k)\varrho_k \oplus \beta_k \Upsilon \varrho_k, ð) \\ &\leq (1-\beta_k) d_w(\varrho_k, ð) + \beta_k d_w(\Upsilon \varrho_k, ð) \\ &\leq (1-\beta_k) d_w(\varrho_k, ð) + \beta_k d_w(\varrho_k, ð) \\ &= d_w(\varrho_k, ð). \end{aligned}$$

Again,

$$\begin{aligned} d_w(\ell_k, ð) &= d_w(\Upsilon\left((1-\alpha_k)\Upsilon\varrho_k \oplus \alpha_k \Upsilon \psi_k\right), ð) \\ &\leq d_w((1-\alpha_k)\Upsilon\varrho_k \oplus \alpha_k \Upsilon\psi_k, ð) \\ &\leq (1-\alpha_k) d_w(\Upsilon\varrho_k, ð) + \alpha_k d_w(\Upsilon\psi_k, ð) \\ &\leq (1-\alpha_k) d_w(\varrho_k, ð) + \alpha_k d_w(\psi_k, ð) \\ &\leq d_w(\varrho_k, ð). \end{aligned} \tag{3}$$

Using the quasi-nonexpansive property and the inequality (3), we have

$$\begin{aligned} d_w(\varrho_{k+1}, ð) &= d_w(\Upsilon \ell_k, ð) \\ &\leq d_w(\ell_k, ð) \\ &\leq d_w(\varrho_k, ð). \end{aligned} \tag{4}$$

Therefore, the sequence $\{d_w(\varrho_k, ð)\}$ is monotone nonincreasing and bounded. Hence, $\lim_{k\to+\infty} d_w(\varrho_k, ð)$ exists for all $ð \in F(\Upsilon)$.

(ii) Assume that,

$$\lim_{k\to+\infty} d_w(\varrho_k, ð) = h > 0.$$

By (i), we get

$$d_w(\psi_k, ð) \leq d_w(\varrho_k, ð)$$

and taking limsup on both sides, we have

$$\limsup_{k\to\infty} d_w(\psi_k, ð) \leq \limsup_{k\to\infty} d_w(\varrho_k, ð) = h. \tag{5}$$

From inequality (4), we know that

$$d_w(\varrho_{k+1}, \eth) \leq d_w(\ell_k, \eth).$$

Taking liminf on both sides, we get

$$h = \lim_{k\to\infty} \inf d_w(\varrho_{k+1}, \eth) \leq \lim_{k\to\infty} \inf d_w(\ell_k, \eth). \tag{6}$$

Using inequalities (5) and (6), together with (3), we have

$$\lim_{k\to\infty} d_w(\ell_k, \eth) = h.$$

From the foregoing inequality and the quasi-nonexpansive property in the Remark 2.10, we also have

$$\begin{aligned} h &= \lim_{k\to\infty} d_w(\ell_k, \eth) = \lim_{k\to\infty} d_w(\Upsilon\left((1-\alpha_k)\,\Upsilon\varrho_k \oplus \alpha_k\Upsilon\psi_k\right), \eth) \\ &\leq \lim_{k\to\infty} d_w\left((1-\alpha_k)\Upsilon\varrho_k \oplus \alpha_k\Upsilon\psi_k, \eth\right) \\ &\leq \lim_{k\to\infty} \left[(1-\alpha_k)d_w(\Upsilon\varrho_k, \eth) + \alpha_k d_w(\Upsilon\psi_k, \eth)\right] \qquad (7) \\ &\leq \lim_{k\to\infty} \left[(1-\alpha_k)d_w(\varrho_k, \eth) + \alpha_k d_w(\psi_k, \eth)\right] \\ &= \lim_{k\to+\infty} d_w\left(\varrho_k, \eth\right) \\ &= h. \end{aligned}$$

Therefore, from the line (7) in the inequality above,

$$\lim_{k\to\infty} d_w\left((1-\alpha_k)\Upsilon\varrho_k \oplus \alpha_k\Upsilon\psi_k, \eth\right) = h.$$

Using Lemma 2.13, we obtain

$$\lim_{k\to+\infty} d_w\left(\Upsilon\varrho_k, \Upsilon\psi_k\right) = 0.$$

Since $\lim_{k\to+\infty} d_w\left(\psi_k, \eth\right) = h$, we have

$$\lim_{k\to\infty} d_w\left((1-\beta_k)\,\varrho_k \oplus \beta_k\Upsilon\varrho_k, \eth\right) = h.$$

Applying Lemma 2.13, we obtain

$$\lim_{k\to+\infty} d_w\left(\varrho_k, \Upsilon\varrho_k\right) = 0.$$

□

We now establish the $\Delta$-convergence result for mappings satisfying the ($RCSC$)-condition in complete $UCW$-hyperbolic spaces.

**Theorem 3.2.** *Let $\tilde{N}, \Omega, \Upsilon$, and $\{\varrho_k\}$ be as in Lemma 3.1. Then, the sequence $\{\varrho_k\}$ $\Delta$-converges to a point in $F(\Upsilon)$.*

*Proof.* By Lemma 3.1, the sequence $\left\{d_w\left(\varrho_k, \psi^\dagger\right)\right\}$ is nonincreasing for every $\psi^\dagger \in F(\Upsilon)$. Hence $\{\varrho_k\}$ is Fejér monotone with respect to $F(\Upsilon)$. Since $F(\Upsilon)$ is closed and convex [16], Proposition 2.8 guarantees that $\{\varrho_k\}$ admits a unique asymptotic center $\omega^\dagger$ with respect to $F(\Upsilon)$. Let $\left\{\varrho_{k_j}\right\}$ be an arbitrary subsequence of $\{\varrho_k\}$. Again by Proposition 2.8, the sequence $\left\{\varrho_{k_j}\right\}$ has a unique asymptotic center $\rho^\dagger$ with respect to $F(\Upsilon)$. Now,

$$\begin{aligned} d_w\left(\varrho_{k_j}, \Upsilon\rho^\dagger\right) &\leq d_w\left(\Upsilon\varrho_{k_j}, \Upsilon\rho^\dagger\right) + d_w\left(\Upsilon\varrho_{k_j}, \varrho_{k_j}\right) \\ &\leq \frac{1}{3}\left[d_w\left(\varrho_{k_j}, \rho^\dagger\right) + d_w\left(\Upsilon\varrho_{k_j}, \rho^\dagger\right) + d_w\left(\varrho_{k_j}, \Upsilon\rho^\dagger\right)\right] + d_w\left(\Upsilon\varrho_{k_j}, \varrho_{k_j}\right) \\ &\leq \frac{1}{3}\left[2d_w\left(\varrho_{k_j}, \rho^\dagger\right) + d_w\left(\Upsilon\varrho_{k_j}, \varrho_{k_j}\right) + d_w\left(\varrho_{k_j}, \Upsilon\rho^\dagger\right)\right] + d_w\left(\Upsilon\varrho_{k_j}, \varrho_{k_j}\right) \end{aligned}$$

and thus, we have

$$d_w\left(\varrho_{k_j}, \Upsilon\rho^\dagger\right) \leq d_w\left(\varrho_{k_j}, \rho^\dagger\right) + 2d_w\left(\Upsilon\varrho_{k_j}, \varrho_{k_j}\right).$$

Using Lemma 3.1 (ii) together with Lemma 2.11, we obtain $\Upsilon\left(\rho^\dagger\right) = \rho^\dagger$. Therefore $\rho^\dagger \in F(\Upsilon)$. Applying Lemma 2.12, it follows that the sequence $\{\varrho_k\}$ $\Delta$-converges to a point of $F(\Upsilon)$. □

Next, we prove a strong convergence result in the compact setting.

**Theorem 3.3.** *Let $(\tilde{N}, d_w, W)$ be a complete UCW-hyperbolic space and $\Omega \neq \emptyset$ is a closed convex and compact subset of $\tilde{N}$. Let $\Upsilon : \Omega \to \Omega$ be a self mapping satisfying (RCSC)-condition and $F(\Upsilon) \neq \emptyset$. Then the sequence $\{\varrho_k\}$ defined by (2) strongly converges to a fixed point of $\Upsilon$.*

*Proof.* From Lemma 3.1, $\lim_{k\to+\infty} d_w(\varrho_k, \Upsilon(\varrho_k)) = 0$. Since $\Omega$ is a compact set, $\{\varrho_k\}$ has a subsequence $\left\{\varrho_{k_j}\right\}$ such that $\left\{\varrho_{k_j}\right\}$ converges strongly to $ð \in \Omega$.

Using Lemma 2.9, we have

$$d_w\left(\varrho_{k_j}, \Upsilon ð\right) \leq 9d_w\left(\varrho_{k_j}, \Upsilon\varrho_{k_j}\right) + d_w\left(\varrho_{k_j}, ð\right).$$

Taking limit as $j \longrightarrow \infty$, we obtain that the sequence $\left\{\varrho_{k_j}\right\}$ converges to $\Upsilon ð$, so $\Upsilon ð = ð$ i.e., $ð \in F(\Upsilon)$. Lemma 3.1 suggests that $\lim_{k\to+\infty} d_w(\varrho_k, ð)$ exists. Therefore, the sequence $\{\varrho_k\}$ converges to $ð$. □

Finally, we establish a strong convergence theorem under Condition $(I)$.

**Theorem 3.4.** *Let $\tilde{N}, \Omega, \Upsilon$, and $\{\varrho_k\}$ be same as in Lemma 3.1. If the mapping $\Upsilon$ has Condition $(I)$, then the sequence $\{\varrho_k\}$ strongly converges to a point in $F(\Upsilon)$.*

*Proof.* By Lemma 3.1, the sequences $\left\{d_w\left(\varrho_k, \psi^\dagger\right)\right\}$ are monotone nonincreasing for all $\psi^\dagger \in F(\Upsilon)$. Thus, the sequence $\{d_w(\varrho_k, F(\Upsilon))\}$ is monotone nonincreasing. Hence, $\lim_{k\to+\infty} d_w(\varrho_k, F(\Upsilon))$ exist. From Lemma 3.1,

$$\lim_{k\to+\infty} d_w(\varrho_k, \Upsilon(\varrho_k)) = 0 \tag{8}$$

Since $\Upsilon$ satisfies Condition $(I)$,

$$d_w(\varrho_k, \Upsilon(\varrho_k)) \geq f(d_w(\varrho_k, F(\Upsilon)))$$

From (8), $\lim_{k\to+\infty} f(d_w(\varrho_k, F(\Upsilon))) = 0$ and hence

$$\lim_{k\to+\infty} d_w(\varrho_k, F(\Upsilon)) = 0. \tag{9}$$

Now, one can verify that the sequence $\{\varrho_k\}$ is Cauchy. For a given $\varepsilon > 0$, from (9), there exists a $k_0 \in \mathbb{N}$ such that for all $k \geq k_0$

$$d_w(\varrho_k, F(\Upsilon)) < \frac{\varepsilon}{4}$$

and

$$\inf\left\{d_w\left(\varrho_{k_0}, \psi^\dagger\right) : \psi^\dagger \in F(\Upsilon)\right\} < \frac{\varepsilon}{4},$$

and there exists $\psi^\dagger \in F(\Upsilon)$ such that

$$d_w\left(\varrho_{k_0}, \psi^\dagger\right) < \frac{\varepsilon}{2}.$$

Therefore, for all $m, k \geq k_0$,

$$d_w(\varrho_{k+m}, \varrho_k) \leq d_w\left(\varrho_{k+m}, \psi^\dagger\right) + d_w\left(\psi^\dagger, \varrho_k\right) \leq 2d_w\left(\varrho_{k_0}, \psi^\dagger\right) < 2\frac{\varepsilon}{2} = \varepsilon,$$

and thus, the sequence $\{\varrho_k\}$ is Cauchy. By the closedness of the set $\Omega$ in complete $UCW$-hyperbolic space $\tilde{N}$, the sequence $\{\varrho_k\}$ converges to a point $ð \in \Omega$. Moreover, since $\lim_{k\to+\infty} d_w(\varrho_k, F(\Upsilon)) = 0$, we obtain $d_w(ð, F(\Upsilon)) = 0$. Since $F(\Upsilon)$ is closed, it follows that $ð \in F(\Upsilon)$. Thus, $ð = \Upsilon(ð)$. Therefore, the sequence $\{\varrho_k\}$ strongly converges to a point in $F(\Upsilon)$. □

The following example illustrates the applicability of Theorem 3.3 to mappings satisfying the ($RCSC$)-condition which are not necessarily mappings satisfying Condition-($C$).

**Example 3.5.** *Let $\Omega = [3,6]$ with the usual metric $d_w(\varrho,\ell) = \left|\varrho - \ell\right|$ and define $\Upsilon : \Omega \to \Omega$ by*

$$\Upsilon(\varrho) = \begin{cases} \frac{\varrho+5}{2} & \text{if } \varrho \neq 6, \\ 5 & \text{if } \varrho = 6. \end{cases}$$

*Then $F(\Upsilon) = \{5\}$. We show that $\Upsilon$ satisfies the ($RCSC$)-condition.*

**Case 1.** *Let $\varrho, \ell \in [3,6)$. Then $\Upsilon(\varrho) = \frac{\varrho+5}{2}$, $\Upsilon(\ell) = \frac{\ell+5}{2}$ and hence*

$$\begin{aligned} d_w(\Upsilon(\varrho),\Upsilon(\ell)) &= \left|\frac{\varrho+5}{2} - \frac{\ell+5}{2}\right| \\ &= \frac{\left|\varrho-\ell\right|}{2} \\ &= \frac{1}{3}\left|\varrho-\ell\right| + \frac{1}{3}\left|\left(\varrho - \frac{\ell+5}{2}\right) - \left(\ell - \frac{\varrho+5}{2}\right)\right| \\ &\leq \frac{1}{3}\left|\varrho-\ell\right| + \frac{1}{3}\left|\varrho - \frac{\ell+5}{2}\right| + \frac{1}{3}\left|\ell - \frac{\varrho+5}{2}\right| \\ &= \frac{1}{3}\left[d_w(\varrho,\ell) + d_w(\Upsilon\varrho,\ell) + d_w(\varrho,\Upsilon\ell)\right]. \end{aligned}$$

*Hence the ($RCSC$)-condition holds.*

**Case 2.** *Let $\varrho = 6$ and $\ell \in [3,6)$. Then $\Upsilon(\varrho) = 5$, $\Upsilon(\ell) = \frac{\ell+5}{2}$ and*

$$\begin{aligned} d_w(\Upsilon(\varrho),\Upsilon(\ell)) &= \left|5 - \frac{\ell+5}{2}\right| \\ &= \left|\frac{\ell-5}{2}\right| \\ &= \frac{1}{3}\left|\frac{3\ell-15}{2}\right| \\ &= \frac{1}{3}\left|\frac{\ell-5}{2} + (\ell-5)\right| \\ &\leq \frac{1}{3}\left|\frac{\ell-5}{2}\right| + \frac{1}{3}|\ell-5| \\ &= \frac{1}{3}\left|(\ell-\varrho) + \left(\varrho - \frac{5+\ell}{2}\right)\right| + \frac{1}{3}|\ell-5| \\ &\leq \frac{1}{3}\left|\ell-\varrho\right| + \frac{1}{3}\left|\varrho - \frac{5+\ell}{2}\right| + \frac{1}{3}|\ell-5| \\ &= \frac{1}{3}\left[d_w(\varrho,\ell) + d_w(\varrho,\Upsilon\ell) + d_w(\Upsilon\varrho,\ell)\right]. \end{aligned}$$

*Thus the ($RCSC$)-condition holds.*

**Case 3.** *Let $\varrho \in [3,6)$ and $\ell = 6$. Similarly, $\Upsilon(\varrho) = \frac{\varrho+5}{2}$, $\Upsilon(\ell) = 5$ and*

$$\begin{aligned}
d_w(\Upsilon(\varrho), \Upsilon(\ell)) &= \left|\frac{\varrho+5}{2} - 5\right| \\
&= \left|\frac{\varrho-5}{2}\right| \\
&= \frac{1}{3}\left|\frac{3\varrho-15}{2}\right| \\
&= \frac{1}{3}\left|\frac{\varrho-5}{2} + (\varrho-5)\right| \\
&\leq \frac{1}{3}\left|\frac{\varrho-5}{2}\right| + \frac{1}{3}\left|\varrho-5\right| \\
&= \frac{1}{3}\left|(\varrho-\ell) + \left(\ell - \frac{5+\varrho}{2}\right)\right| + \frac{1}{3}\left|\varrho-5\right| \\
&\leq \frac{1}{3}\left|\varrho-\ell\right| + \frac{1}{3}\left|\ell - \frac{5+\varrho}{2}\right| + \frac{1}{3}\left|\varrho-5\right| \\
&= \frac{1}{3}\left[d_w(\varrho,\ell) + d_w(\Upsilon\varrho,\ell) + d_w(\varrho,\Upsilon\ell)\right].
\end{aligned}$$

*Therefore $\Upsilon$ satisfies the (RCSC)-condition.*

*Moreover, $\Upsilon$ does not satisfy the Condition-(C). Indeed, take $\varrho = 5.7$ and $\ell = 6$. Then $\Upsilon(\varrho) = 5.35$, $\Upsilon(\ell) = 5$. Moreover,*

$$\frac{1}{2}d_w(\varrho, \Upsilon(\varrho)) = \frac{1}{2}(0.35) = 0.175 < 0.3 = d_w(\varrho,\ell),$$

*but*

$$d_w(\Upsilon(\varrho), \Upsilon(\ell)) = 0.35 > 0.3 = d_w(\varrho,\ell).$$

*Hence $\Upsilon$ satisfies the (RCSC)-condition but does not satisfy Condition-(C).*

*Since $\Omega$ is closed, convex and compact, all assumptions of Theorem 3.3 are satisfied. Therefore, the sequence $\{\varrho_k\}$ generated by the iterative process (2) strongly converges to the fixed point 5.*

*To compare the convergence behavior of the considered iterative schemes, we apply the M, M* and K-iteration processes to Example 3.5 with $\varrho_1 = 4$ and $\alpha_k = \beta_k = 0.5$. The numerical results are presented in Table 1. It can be observed that all three methods converge to the fixed point $ð = 5$. Furthermore, the K-iteration process converges to* *the fixed point at a faster rate than the the other two methods.*

| $k$ | $M$ | $M^*$ | $K$ |
|---|---|---|---|
| 1 | 4.0000000000 | 4.0000000000 | 4.0000000000 |
| 2 | 4.8125000000 | 4.8281250000 | 4.8906250000 |
| 3 | 4.9648437500 | 4.9704589844 | 4.9880371094 |
| 4 | 4.9934082031 | 4.9949226379 | 4.9986915588 |
| 5 | 4.9987640381 | 4.9991273284 | 4.9998568892 |
| 6 | 4.9997682571 | 4.9998500096 | 4.9999843473 |
| 7 | 4.9999565482 | 4.9999742204 | 4.9999982880 |
| 8 | 4.9999918528 | 4.9999955691 | 4.9999998127 |
| 9 | 4.9999984724 | 4.9999992384 | 4.9999999795 |
| 10 | 4.9999997136 | 4.9999998691 | 4.9999999978 |
| 11 | 4.9999999463 | 4.9999999775 | 4.9999999998 |
| 12 | 4.9999999899 | 4.9999999961 | 5.0000000000 |
| 13 | 4.9999999981 | 4.9999999993 | 5.0000000000 |
| 14 | 4.9999999996 | 4.9999999999 | 5.0000000000 |
| 15 | 4.9999999999 | 5.0000000000 | 5.0000000000 |

*TABLE 1. Comparison of the M, $M^*$ and K-iteration processes*

*The computations were performed using a MATLAB R2023b program.*

## 4. Applications

Let $\tilde{N} = L^2\left([0,1], \mathbb{R}\right)$ endowed with the norm

$$\left\|\varrho\right\|_2 = \left(\int_0^1 \left|\varrho(t)\right|^2 dt\right)^{1/2}$$

and metric

$$d_w(\varrho, \ell) = \left\|\varrho - \ell\right\|_2 .$$

Define convex structure by

$$W(\varrho, \ell, \mu) = (1-\mu)\varrho + \mu\ell, \ \mu \in [0,1].$$

Then $(\tilde{N}, d_w, W)$ is a complete $UCW$-hyperbolic space. Let

$$\Omega = \left\{\varrho \in \tilde{N} : \left\|\varrho\right\|_2 \leq R\right\},$$

where $R > 0$. Then $\Omega$ is a nonempty closed convex subset of $\tilde{N}$.

Consider the nonlinear Fredholm integral equation

$$\varrho(t) = g(t) + \int_0^1 K(t, s, \varrho(s))ds, \ t \in [0,1]. \tag{10}$$

Assume that $g \in L^2\left([0,1], \mathbb{R}\right)$ and $K : [0,1] \times [0,1] \times \mathbb{R} \longrightarrow \mathbb{R}$ satisfies the following conditions:

(i) $K(t, s, u)$ is measurable with respect to $t$ and $s$ for each $u \in \mathbb{R}$,

(ii) there exists a constant $q \in (0, \frac{1}{3}]$ such that

$$\left|K(t, s, u) - K(t, s, v)\right| \leq q\left|u - v\right|$$

for all $t, s \in [0,1]$ and $u, v \in \mathbb{R}$,

(iii)

$$\|g\|_2 + \left(\int_0^1 \left(\int_0^1 |K(t,s,0)|\, ds\right)^2 dt\right)^{1/2} + qR \leq R.$$

Define mapping $\Upsilon : \Omega \to \Omega$ by

$$(\Upsilon \varrho)(t) = g(t) + \int_0^1 K(t,s,\varrho(s))ds. \tag{11}$$

By condition (iii), $\Upsilon$ maps $\Omega$ into itself. For $\varrho, \ell \in \Omega$ using condition (ii), we obtain

$$\begin{aligned}
d_w(\Upsilon\varrho, \Upsilon\ell) &= \left(\int_0^1 \left(\int_0^1 \left|K(t,s,\varrho(s)) - K(t,s,\ell(s))\right| ds\right)^2 dt\right)^{1/2} \\
&\leq q\left(\int_0^1 \left(\int_0^1 \left|\varrho(s) - \ell(s)\right| ds\right)^2 dt\right)^{1/2} \\
&\leq q\left\|\varrho - \ell\right\|_2 \\
&= q d_w(\varrho, \ell).
\end{aligned}$$

Since $q \leq \frac{1}{3}$, whenever $\frac{1}{2}d_w(\varrho, \Upsilon\varrho) \leq d_w(\varrho, \ell)$, we have

$$d_w(\Upsilon\varrho, \Upsilon\ell) \leq q d_w(\varrho, \ell) \leq \frac{1}{3}\left[d_w(\varrho, \ell) + d_w(\Upsilon\varrho, \ell) + d_w(\varrho, \Upsilon\ell)\right].$$

Thus, $\Upsilon$ satisfies the $(RCSC)$-condition.

Moreover,

$$d_w(\varrho, \Upsilon\varrho) \geq d_w(\varrho, ð) - d_w(\Upsilon\varrho, \Upsilonð) \geq (1-q)d_w(\varrho, ð),$$

where $ð \in F(\Upsilon)$. Hence $\Upsilon$ satisfies Condition $(I)$ with $f(r) = (1-q)r$. Therefore, all assumptions of Theorem 3.4 are satisfied. Consequently, the sequence $\{\varrho_k\}$ generated by (2) converges strongly to a fixed point $ð \in \Omega$. Since $ð = \Upsilonð$, it follows that $ð$ is a solution of the nonlinear Fredholm integral equation (10).

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