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A New Exponentially Fast Quantum Algorithm for Searching Target in the Unstructured Database

Dhananjay P. Mehendale

Department of Physics, Savitribai Phule University of Pune, Pune, India-411007.

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Abstract: We propose a fresh new idea of cutting the size of the search space (database) to half at each iteration and thus achieve an efficient quantum search algorithm with complexity $O(LogN)^2$. This is exponential speedup over Grover's algorithm. Grover's quantum algorithm [6] has been proven to be optimal [1,2,7]. It has been shown that for any number of oracle lookups up to about $\pi/4\sqrt{N}$, Grover's quantum searching algorithm will give the maximal possible probability of finding the desired target [1]. Therefore, as long as the entire setup used for the Grover's algorithm remains the same i.e. (i) the database to which the unique target belongs remains the same and (ii) we consider to invoke only those oracles which give value 1 for exactly one unique input called the target we cannot do any better. In the quantum algorithm that we propose here we do not change (ii) but we manage to change (i) i.e. we manage to change the search space (database) after each iteration, in fact we manage to reduce its size by half by excluding that half part of the search space having no (or zero) overlap with the target state by making use of the inner product [3] of the the target state and the register corresponding half part of the database under consideration. We show that this simple change in the setup enables us to achieve exponential speedup over Grover's quantum algorithm. In this new quantum search algorithm the amplitude of the target state is not amplified directly as is done in Grover's algorithm instead the proposed algorithm reduces the size of the search space by half in each iterative step which indirectly amplifies (doubles) the amplitude of the target state during each iteration and attains the maximal amplitude, ≈ 1, for it in $LogN$ iterations.

Keywords: Inner Product, Iterative Reduction of Search space, Quantum Binary Search

1 Introduction

The proposed algorithm prepares two quantum registers of equal size which are made up of equally weighted superposition of $\frac{N}{2}$ computational basis states. The first register contains the data elements x such that $0 \leq x \leq (\frac{N}{2}) - 1$ and the second register contains the data elements x such that $\frac{N}{2} \leq x \leq (N-1)$. This algorithm requires in all $LogN$ iterative steps and in each such iterative step one requires to determine the inner product of the target state and the quantum state representing the database. The quantum algorithm for finding inner product of two N dimensional vectors has complexity equal to $LogN$ [3]. Thus the overall complexity of our proposed quantum algorithm turns out to be $O(LogN)^2$. Exactly one of these two quantum registers will have nonzero overlap with the target state, $|t\rangle$, belonging to the original database and to be searched. The register between the two newly prepared registers having a nonzero overlap with the target state is determined at each iterative step by taking the inner product of the target state with any one of these two quantum registers in $LogN$ time [3].

The algorithm for search in the unstructured database that is proposed here is nothing but "a quantum binary search algorithm'. This iterative algorithm divides the given database at each iterative step into two equal sized databases in terms of two quantum registers and by finding inner product in $LogN$ time [3] of any one these quantum registers with the implicitly known target state one finds out that quantum register between these two quantum registers which has nonzero overlap with the target state i.e. one finds out that quantum register between these two quantum registers to which the target state belongs. In nutshell this new quantum algorithm proceeds as follows: One prepares two quantum registers of equal size such that only one of them contains the target state $|t\rangle$, and the other does not. In short one carries out the construction of two quantum registers such that only one out of these two quantum registers will contain

* Corresponding author e-mail: dhananjay.p.mehendale@gmail.com

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the target state, $|t\rangle$. One then evaluates the inner product of this target state, $|t\rangle$, with any one of these two newly constructed quantum registers. Now the value of this inner product taken with any one of these two registers will be zero if the target state $|t\rangle$ does not belong to that register and on the other hand this value of the inner product will be nonzero and will be equal to $\sqrt{\frac{2}{N}}$ at the first iteration (and will increase to value equal to $\sqrt{\frac{2^k}{N}}$ at the $k-$th iteration) if the target state $|t\rangle$ belongs to that register. Thus at each iterative step the value of the inner product determines to which of these quantum registers the target state, $|t\rangle$, belongs. This quantum register to which target state, $|t\rangle$, belongs is then used to further constructing two new quantum registers and the same procedure is repeated iteratively. By iterating these steps for $LogN$ time one will see that one directly arrives at the desired target state,$|t\rangle$, and thus the search for finding out the target state, $|t\rangle$, is complete.

The classical generate-and-test algorithm [4] or the well-known quantum algorithm for unstructured database search [6] though can be easily adapted to solve the NP-Complete problems still the computational complexity of these algorithms after the adaptations remains exponential in order as the exponent for quantum algorithm [6] changes only to one-half times the exponent for classical algorithm. But for the new quantum algorithm proposed in this paper the exponent becomes the polynomial of logarithm of the exponent for the classical algorithm. Therefore, the quantum algorithm proposed in this paper reduces the computational complexity of NP-Complete problems to polynomial order. If N is the size of the unstructured database then we show that we can attain the desired target in just $LogN$ steps and each such step requires $LogN$ time. To attain the desired target the best known generate-and-test type classical algorithm and quantum algorithm [6] for unstructured search requires roughly $\frac{N}{2}$ steps and $\sqrt{(N)}$ steps respectively. This implies that only quadratic speedup is achievable by the quantum algorithm [6] over the classical algorithm. Though such speedup is quite good still it is not good enough as it does not tame the problems with exponential complexity.

2 The Unstructured Search Problem and the Oracle:

A formal statement of unstructured search problem is as follows: Consider a search problem that requires to find a particular element of the database. Given a set (database) containing $N = 2^n$ candidates, and suppose these N candidates are labeled by indices, x in the range $0 \leq x \leq (N-1)$ and that the index of the sought after target item is $x = t$. Let there be a computational oracle, or "black-box function", $f_t(x)$, that when presented with an index x can pronounce on whether or not it is the index of the target. Specifically, $f_t(x)$ is defined such that $f_t(x) = 1$ if $x = t$ and $f_t(x) = 0$ if $x \neq t$ where "1" stands for "YES" and "0" stands for "NO". The search problem is unstructured because there is no discernible pattern to the values of $f_t(x)$ to provide any guidance in finding $x = t$. Our job is to find index, $x = t$, using fewest calls to the oracle $f_t(x)$. Oracle is nothing but a fictitious mathematical device that allows one to estimate the computational cost of some algorithm measured in the units of the "number of calls to this oracle", required to reach the solution. "Oracle" or "black-box function" or "knowledge holder" are synonyms, and if we consider for example the problem of finding name given telephone number then what is the oracle? The 'oracle' in this case is the 'telephone directory' itself [4].

We now express the search problem in quantum mechanical language. A quantum analog of the bag of indices, the so called "quantum register", is made up of equally weighted superposition of all the computational basis states, $|x\rangle$ such that the index x is in the range $0 \leq x \leq (N-1)$, i.e. the quantum state $|\Psi\rangle = \frac{1}{\sqrt{N}} \sum_{x=0}^{(N-1)} |x\rangle$. Thus, the bag of all indices can be looked upon as a wave function $|\Psi\rangle$ given above. Let us suppose that $N = 2^n$, therefore, using binary representation for all the indices in the bag we can express the quantum register representing bag of indices as

$$|\Psi\rangle = \frac{1}{\sqrt{N}} \sum_{i_1,i_2,\cdots,i_n=0}^{1} |i_1 i_2 \cdots i_n\rangle$$

To prepare such quantum register is in fact a very easy task. One just needs to take as a starting state a tensor product of n number of kets, $|0\rangle$, and then apply Hadamard operator, H, on each of these kets, $|0\rangle$, in the tensor product. Thus,

$$|\Psi\rangle = \frac{1}{\sqrt{N}} \sum_{i_1,i_2,\cdots,i_n=0}^{1} |i_1 i_2 \cdots i_n\rangle = H^{\otimes n}|00\cdots 0\rangle$$

The implicitly known target state is $|t\rangle = |t_1 t_2 t_3 \cdots t_n\rangle$, where each $t_i \in \{0,1\}$, can be prepared using the oracle, $f_t(x)$, which takes value 1 when $x = t$ and 0 when $x \neq t$. We can express the target state, $|t\rangle$ corresponding to index $x = t$ by using the expression for the wave function, $|\Psi\rangle$, namely,

$$|\Psi\rangle = \frac{1}{\sqrt{N}} \sum_{x=0}^{(N-1)} |x\rangle$$

in the following two equivalent forms:

$$|t\rangle = \frac{\sqrt{N}}{2}[1 - (-1)^{f_t(x)}]|\Psi\rangle$$

or,

$$|t\rangle = \frac{\sqrt{N}}{2}[1 - [1 - 2|t\rangle\langle t|]]|\Psi\rangle$$

We now divide the elements in the bag containing indices x, such that $0 \leq x \leq (N-1)$, into two bags such that the

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first bag will contain first half indices, i.e. all those indices, x, such that $0 \leq x \leq (\frac{N}{2}-1)$, and the second bag will contain remaining second half indices, i.e. all those indices, x, such that $\frac{N}{2} \leq x \leq (N-1)$. It is easy to achieve this by constructing these bags in terms of two quantum registers, $|\Psi_0\rangle$ and $|\Psi_1\rangle$ as follows:

$$|\Psi_0\rangle = |0\rangle \otimes H^{\otimes(n-1)}|00\cdots0\rangle$$

and,

$$|\Psi_1\rangle = |1\rangle \otimes H^{\otimes(n-1)}|00\cdots0\rangle$$

Note that the ket $|00\cdots0\rangle$ in the above expressions for $|\Psi_0\rangle$ and $|\Psi_1\rangle$ is of length $(n-1)$, i.e. a computational basis state in $2^{(n-1)}$ dimensional Hilbert space, while the whole $|\Psi_0\rangle$ and $|\Psi_1\rangle$ are obviously the quantum states in $N = 2^n$ dimensional Hilbert space. Also, $|\Psi_0\rangle$ represents the bag that contains all those indices, x, such that $0 \leq x \leq (\frac{N}{2}-1)$, and $|\Psi_1\rangle$ and $|\Psi_1\rangle$ represents the bag that contains all those indices, x, such that $\frac{N}{2} \leq x \leq (N-1)$. The idea behind our new quantum algorithm in simple terms is to divide "the quantum bag which contains the target state, $|t\rangle$," at each iterative step into two separate bags of equal size such that now the target state will belong to some one and only one of these two bags which now have become equal to half the size of the original bag and then to determine by taking inner product [3] of any one quantum register representing these bags with the target state, $|t\rangle$, we determine to which of these newly prepared quantum registers this target state, $|t\rangle$, belongs. Thus we manage to reduce the size of the bag that contains the target state in each of the iterations to half of its size at the previous stage. By proceeding along these lines finally the bag that contains the target state will become of size one, i.e. it will become the target state, $|t\rangle$, itself. Thus, we first begin with the bag represented in terms of the quantum register, $|\Psi\rangle$, this original quantum register contains basic computational states corresponding to all numbers from 0 to $(N-1)$, and so it also contains the target state, $|t\rangle$.

3 The New Quantum Search Algorithm:

We now proceed systematically with our new quantum algorithm through precise steps as follows:

(i) Construct quantum state, $|\Psi\rangle$ say, representing the bag of all indices x, such that $0 \leq x \leq (N-1)$. Namely,

$$|\Psi\rangle = \frac{1}{\sqrt{N}} \sum_{i_1,i_2,\cdots,i_n=0}^{1} |i_1 i_2 \cdots i_n\rangle = H^{\otimes n}|00\cdots0\rangle$$

Let $N = 2^n$. Since, $t \in [0,(2^n-1)]$, therefore, $|t\rangle$ will be present as a computational basis state in the quantum register $|\Psi\rangle$, i.e. certainly, $\langle t|\Psi\rangle = \frac{1}{\sqrt{N}} \neq 0$.

(ii) Since $t \in [0,(2^n-1)]$, we divide the indices in this quantum register into two parts of equal size and prepare two new quantum registers. This is done in equivalent terms as follows. Construct two quantum registers $|\Psi_0\rangle$ and $|\Psi_1\rangle$ representing these two bags such that $|\Psi_0\rangle$ will represent the bag that contains all those indices, x such that $0 \leq x \leq (2^{(n-1)}-1)$ and $|\Psi_1\rangle$ will represent the bag that contains all those indices, x, such that $2^{(n-1)} \leq x \leq (2^n-1)$. Thus, we have

$$|\Psi_0\rangle = |0\rangle \otimes H^{(n-1)}|00\cdots0\rangle$$

and,

$$|\Psi_1\rangle = |1\rangle \otimes H^{(n-1)}|00\cdots0\rangle$$

Clearly, we have either $|t\rangle \in |\Psi_0\rangle$ or $|t\rangle \in |\Psi_1\rangle$.

(iii) Take inner product of the implicitly known target state $|t\rangle$, expressed above in two equivalent forms with any one of these two newly created quantum registers representing two bags of indices, namely, $|\Psi_0\rangle$ and $|\Psi_1\rangle$ given above.

Case (a): Without loss of generality, if $\langle t|\Psi_0\rangle = \sqrt{\frac{2}{N}} \neq 0$ then clearly we can infer that $|t\rangle \in |\Psi_0\rangle$, i.e. $t \in [0,(2^{(n-1)}-1)]$.

Case (b): Without loss of generality, if $\langle t|\Psi_0\rangle = 0$ (and so $\langle t|\Psi_1\rangle = \sqrt{\frac{2}{N}} \neq 0$,) then clearly we can infer that $|t\rangle \in |\Psi_1\rangle$, i.e. $t \in [2^{(n-1)},(2^n-1)]$.

Suppose Case (a) is true: Then $t \in [0,(2^{(n-1)}-1)]$, we then divide this bag of indices into two equal parts and put them into two new bags. This is done in equivalent terms as follows. Construct two new quantum registers $|\Psi_{00}\rangle$ and $|\Psi_{01}\rangle$ representing these two new quantum bags such that $|\Psi_{00}\rangle$ will represent the bag that contains computational basis states corresponding to all those indices, x, such that $0 \leq x \leq (2^{(n-2)}-1)$ and $|\Psi_{01}\rangle$ will represent the bag that contains computational basis states corresponding to all those indices, x, such that $2^{(n-2)} \leq x \leq (2^{(n-1)}-1)$. Thus, we have

$$|\Psi_{00}\rangle = |0\rangle|0\rangle \otimes H^{(n-2)}|00\cdots0\rangle$$

and,

$$|\Psi_{01}\rangle = |0\rangle|1\rangle \otimes H^{(n-2)}|00\cdots0\rangle$$

This further implies that either $|t\rangle \in |\Psi_{00}\rangle$ or $|t\rangle \in |\Psi_{01}\rangle$.

Suppose case (b) is true: Then $t \in [2^{(n-1)},2^n-1]$, we divide this bag of indices into two equal parts and put them into two new bags. This is done in equivalent terms as follows. Construct two new quantum registers $|\Psi_{10}\rangle$ and $|\Psi_{11}\rangle$ representing these two new bags such that $|\Psi_{10}\rangle$ will represent the quantum bag that contains computational basis states corresponding to all those indices, x, such that $2^{(n-1)} \leq x \leq 2^{(n-1)} + (2^{(n-2)}-1)$ and $|\Psi_{11}\rangle$ will represent the quantum bag that contains computational basis states corresponding to all those indices, x, such that $(2^{(n-1)} + 2^{(n-2)}) \leq x \leq (2^n-1)$. Thus, we have

$$|\Psi_{10}\rangle = |1\rangle|0\rangle \otimes H^{(n-2)}|00\cdots0\rangle$$

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and,

$$|\Psi_{11}\rangle = |1\rangle|1\rangle \otimes H^{(n-2)}|00\cdots 0\rangle$$

This further implies that either $|t\rangle \in |\Psi_{10}\rangle$ or $|t\rangle \in |\Psi_{11}\rangle$.

(iv) As is done in (iii), by taking inner product of the target state $|t\rangle$, now with $|\Psi_{00}\rangle$ or $|\Psi_{01\rangle}$ when case (a) is true, or with $|\Psi_{10}\rangle$ or $|\Psi_{11}\rangle$ when case (b) is true we determine to which quantum bag represented by these newly constructed quantum registers the target state, $|t\rangle$, belongs, i.e. to which quantum bag represented by these two newly constructed quantum registers the target state, $|t\rangle$, has a nonzero overlap. We continue on these lines with dividing, each time the correct quantum bag, to which the target state belongs, into two separate new quantum bags till (assuredly) the size of the correct quantum bag (that has nonzero overlap with the target state, $|t\rangle$,) will reduce to the bag containing just one entry, i.e. the bag will finally reduce to the target state, $|t\rangle$ itself.

4 Remarks:

Remark 1: It is clear to see that as the algorithm proceeds we get at each iteration the bag containing proper range of indices to which target index belongs, i.e. we get during each of the iterations a proper quantum bag reduced to half in size, in terms of a quantum state which has nonzero overlap with target state. Thus, as we proceed at an intermediate stage we reach at a quantum register, say,

$$|\Psi_{i_1 i_2 \cdots i_k}\rangle = |i_1 i_2 \cdots i_k\rangle \otimes H^{\otimes(n-k)}|00\cdots 0\rangle_{(n-k)}$$

which has nonzero overlap with the target state, $|t\rangle$. We then divide this quantum register into two new quantum registers, i.e. we construct two new quantum registers out of which only one will have nonzero overlap with the target state, $|t\rangle$, to be determined by taking inner product with any one of these two newly prepared quantum registers. Thus, the new quantum registers constructed from the earlier reached above mentioned quantum register will be

$$|\Psi_{i_1 i_2 \cdots i_k 0}\rangle = |i_1 i_2 \cdots i_k\rangle \otimes |0\rangle \otimes H^{\otimes(n-k-1)}|00\cdots 0\rangle_{(n-k-1)}$$

and,

$$|\Psi_{i_1 i_2 \cdots i_k 1}\rangle = |i_1 i_2 \cdots i_k\rangle \otimes |1\rangle \otimes H^{\otimes(n-k-1)}|00\cdots 0\rangle_{(n-k-1)}$$

for newly checking the overlap with the target state, $|t\rangle$.

Remark 2: It is interesting to see that the amplitude of each state in the equally weighted superposition of states (including target state) is initially equal to $\frac{1}{\sqrt{N}}$. This state represents the initial quantum bag. After first iteration the size of the quantum bag reduces to half, after each next iteration the size of the quantum bag reduces to half of the earlier size, and the story continues and finally this size reduces to one, i.e. finally (at the $n-$th iteration) the quantum bag will contain only the target state itself! Therefore, after first iteration the amplitude of each state in the equally weighted superposition of states containing the target state becomes $\sqrt{\frac{2}{N}}$. The amplitude of each state including target state in the superposition changes in the successive iterations as follows:

$$\frac{1}{\sqrt{N}} \rightarrow \sqrt{\frac{2}{N}} \rightarrow \sqrt{\frac{2^2}{N}} \rightarrow \cdots \rightarrow \sqrt{\frac{2^n}{N}} = 1$$

Remark 3: It is clear to see that in $n = LogN$ iterations we will attain the target state, i.e. the final quantum bag (register), after carrying out $n = LogN$ iterations, will contain only the target state $|t\rangle$ itself which will lead to getting the value of inner product with the target state equal to unity.

Remark 4: As far as the value of inner product is concerned we are only interested to know whether it is zero or nonzero, and we are not interested in its exact value. Therefore we can use the existing quantum algorithm [3] to evaluate the inner product with complexity $O(LogN)$. Since our new quantum algorithm requires $LogN$ number of steps to reach the desired target state and each iterative step requires to find out one inner product which takes $LogN$ time therefore, our new quantum search algorithm is of the order $O(LogN)^2$.

Remark 5: For a typical NP-Complete problem in which one has to find an assignment of one of the b values to each of the C variables, the number of candidate solutions, $N = b^C$, grows exponentially with C. Hence, the classical algorithm would therefore take time of the order, $O(b^C)$ to find desired solution for NP-Complete problems [4,5]. For example minimum weight Hamiltonian circuit among the all possible Hamiltonian circuits as a solution for the traveling salesman problem, whereas the Grover quantum algorithm [6] would take a time of the order, $O(b^{\frac{C}{2}})$. But from the above obtained complexity of the order $O(LogN)^2$ for solving the problem of unstructured search that we get for our quantum search algorithm it is easy to check that our quantum algorithm for finding minimum weight Hamiltonian circuit among the all possible Hamiltonian circuits as a solution for the traveling salesman problem will then takes time of the order, $O(b^{(LogC)^2})$. Thus an impressive (exponential) speedup over existing classical or quantum algorithm is made possible by our new search algorithm. We thus have managed using our new quantum search algorithm a polynomial time complexity for solving NP-Complete problems.

5 An example

Let the bag of indices contains numbers $\{0,1,\cdots,15\}$ and let the target element, $t = 11$. We begin with the corresponding quantum register, $|\Psi\rangle$, namely, $|\Psi\rangle = H^{\otimes 4}|0000\rangle$ which contains the target state, $|t\rangle = |1011\rangle$. We now follow the steps of the algorithm:

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Clearly, $\langle t|\Psi\rangle = \frac{1}{4} \neq 0$, therefore, we now divide the quantum register represented by $|\Psi\rangle$ into two quantum registers, represented by $|\Psi_0\rangle$ and $|\Psi_1\rangle$, where $|\Psi_0\rangle = |0\rangle \otimes H^{\otimes 3}|000\rangle$, and $|\Psi_1\rangle = |1\rangle \otimes H^{\otimes 3}|000\rangle$. Clearly, $\langle t|\Psi_1\rangle = \frac{1}{\sqrt{2^3}} \neq 0$, therefore, we further divide quantum register represented by $|\Psi_1\rangle$ into two quantum registers, represented by $|\Psi_{10}\rangle$ and $|\Psi_{11}\rangle$, where

$$|\Psi_{10}\rangle = |1\rangle \otimes |0\rangle \otimes H^{\otimes 2}|00\rangle,$$

and

$$|\Psi_{11}\rangle = |1\rangle \otimes |1\rangle \otimes H^{\otimes 2}|00\rangle.$$

Clearly, $\langle t|\Psi_{10}\rangle = \frac{1}{2} \neq 0$, therefore, we further divide quantum bag represented by $|\Psi_{10}\rangle$ into two new quantum registers, represented by $|\Psi_{100}\rangle$ and $|\Psi_{101}\rangle$, where

$$|\Psi_{100}\rangle = |1\rangle \otimes |0\rangle \otimes |0\rangle \otimes H|0\rangle,$$

and

$$|\Psi_{101}\rangle = |1\rangle \otimes |0\rangle \otimes |1\rangle \otimes H|0\rangle.$$

Clearly, $\langle t|\Psi_{101}\rangle = \frac{1}{\sqrt{2}} \neq 0$, therefore, we further divide quantum register represented by $|\Psi_{101}\rangle$ into two new quantum registers, represented by $|\Psi_{1010}\rangle$ and $|\Psi_{1011}\rangle$, where

$$|\Psi_{1010}\rangle = |1\rangle \otimes |0\rangle \otimes |1\rangle \otimes |0\rangle,$$

and

$$|\Psi_{1010}\rangle = |1\rangle \otimes |0\rangle \otimes |1\rangle \otimes |1\rangle.$$

Clearly, $\langle t|\Psi_{1011}\rangle \neq 0$, and in fact $\langle t|\Psi_{1011}\rangle = 1$, and as expected the quantum bag has reduced into the target state itself. Thus, we have managed to reach to the desired target state, $|t\rangle = |1011\rangle$, present in the given database in terms of the initial quantum register containing target state as well as all the other computational basis states as the superposition state, $|\Psi\rangle = H^{\otimes 4}|0000\rangle$.

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