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Soliton Solutions of Wu–Zhang System of Evolution Equations

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Abstract: This study secures optical Solitons to Wu–Zhang system of evolution equations that illustrates (1+1)-dimensional dispersive long wave. Four methods called Csch method; Tan-Cot method, Extended Tanh-Coth method, and Modified Simple Equation Method are utilized to retrieve solutions of this equation.

Keywords: Soliton Solution, Csch method, Tan-Cot method, Extended Tanh-Coth method, and Modified Simple Equation Method.

1 Introduction

The solution of nonlinear partial differential equations has enjoyed on the last 30 years from both theoretical and practical points of view. In this context, it is worth mentioning that there are various methods that have been successfully implemented to analyze the physical phenomena and got exact solutions that facilitating the test of the numerical schemes [1].

Different powerful methods were used, such as Tanh-Sech method [2,3,4],extended Tanh-Coth method [5,6,7], hyperbolic function method [8,9], Jacobi elliptic function expansion method [10], F-expansion method [11], and the First Integral method [12], tan- cot function method [13], Sine-Cosine function method [14,15], Modified Simple Equation Method (MSEM) [16].

2 The Governing Wu–Zhang Systems of Equations

The Wu–Zhang system given by [17]:

$$u_t(x,t) + u\,u_x + v_x = 0 \tag{1}$$

$$v_t(x,t) + v\,u_x + u\,v_x + \tfrac{1}{3}u_{xxx} = 0 \tag{2}$$

The governing equations describe the wave on shallow waters. Zheng et al [18] constructed exact solutions when applied the generalized extended Tanh function method. Mirzazadeh et al [19] fined the solitary wave, singular solitary wave, and shock wave solutions. Inc et al [20] applied the extended Tanh and Hirota methods to find the singular solitary wave, multi soliton, and periodic solutions.

3 Methodology

In this study, we applied Csch method, Tan-Cot function method, Extended Tanh-Coth function method, and the Modified Simple Equation Method to extract exact traveling wave solutions of Wu–Zhang system.
Let consider the NLPdE in the form [17]

Here the travelling wave solution of non-linear partial differential equation Eq.(1-2) is $u_1(x,t)$, and $u_2(x,t)$. We use the transformations $u(x,y,t) = f(\xi)$ where $\xi = k(x - \lambda t)$ that enable us to use the following changes,

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$$\frac{\partial}{\partial t}(.) = -\lambda k \frac{d}{d\xi}(.), \frac{\partial}{\partial x}(.) = k \frac{d}{d\xi}(.), \frac{\partial^2}{\partial x^2}(.) = k^2 \frac{d^2}{d\xi^2}(.) \quad (4)$$

Therefore system of Eq.(1-2) is transfer to non-linear ordinary differential equation

$$Q_1(f, f', f'', f''', \dots\dots) = 0 \quad (5\text{-a})$$

$$Q_2(f, f', f'', f''', \dots\dots) = 0 \quad (5\text{-b})$$

The ordinary differential equations in (5-a-b) are then integrated as long as all terms contain derivatives, where we neglect the integration constants.

3.1. The Tan-Cot Function Method

The solutions can be expressed in the form [13]:

$$u(\xi) = A \tan^{\beta}(\mu\xi), \quad |\xi| \le \pi/2\mu \quad \text{or} \quad (6)$$

$$u(\xi) = A \cot^{\beta}(\mu\xi), \quad |\xi| \le \pi/2\mu \quad (7)$$

where A, μ, β parameters to be determined,.

We use

$$f(\xi) = A\, tan^{\beta}(\mu\xi)$$

$$f' = A\beta\mu\left[tan^{(\beta-1)}(\mu\xi) + tan^{(\beta+1)}(\mu\xi)\right]$$

$$f'' = A\beta\mu^2\left[(\beta-1)tan^{(\beta-2)}(\mu\xi) + 2\beta\, tan^{\beta}(\mu\xi) + (\beta+1)tan^{(\beta+2)}(\mu\xi)\right] \quad (8)$$

And their derivatives, or use:

$$u(\xi) = A\, cot^{\beta}(\mu\xi)$$

$$u' = -A\beta\mu\left[cot^{(\beta-1)}(\mu\xi) + cot^{(\beta+1)}(\mu\xi)\right]$$

$$u'' = A\beta\mu^2\left[(\beta-1)cot^{(\beta-2)}(\mu\xi) + 2\beta\, cot^{\beta}(\mu\xi) + (\beta+1)cot^{(\beta+2)}(\mu\xi)\right] \quad (9)$$

We collect all terms with the same power in tank (μξ) or cotk(μξ) and set to zero their coefficients to get a system of algebraic equations with the unknowns A, β, μ and solve the algebraic system of equations.

3.2. Csch function method

Many solutions of evolution equations can be considered in the form [21]:

$$V(\xi) = A\, \mathrm{csch}^{\tau}(\mu\,\xi) \quad (10)$$

and their derivative.

$$V'(\xi) = -A\,\tau\,\mu\,\mathrm{csch}^{\tau}(\mu\xi)\,.\coth(\mu\xi)$$

$$V''(\xi) = A\,\tau\,\mu^2\left((\tau+1)\,\mathrm{csch}^{\tau+2}(\mu\xi) + \tau\,\mathrm{csch}^{\tau}(\mu\xi)\right) \quad (11)$$

where A, μ, τ parameters to be determined, μ is the wave number.

3.3. Tanh-Coth Method

Assume, the new independent variable [5]

$$Y = \tanh(\xi) \quad (12)$$

Then the change of variables will be:

$$\frac{dV}{d\xi} = (1-Y^2)\frac{dV}{dY}$$

$$\frac{d^2V}{d\xi^2} = -2Y(1-Y^2)\frac{dV}{dY} + (1-Y^2)^2\frac{d^2V}{dY^2} \quad (13)$$

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Then the solution is expressed in the form

$$V(\xi) = \sum_{i=0}^{P} a_i\, Y^i + \sum_{i=1}^{P} b_i\, Y^{-i} \tag{14}$$

where the parameter p can be found by balancing the highest-order linear term with the nonlinear terms in the reduced Equation.

3.4. Modified Simple Equation Method (MSEM)

Suppose that solution of evolution equations can be considered by a polynomial in [16]:

$$u(\xi) = \sum_{i=0}^{m} A_i \left(\frac{\psi_\xi}{\psi}\right)^i,\ A_m \neq 0 \tag{15}$$

where m is a positive integer that determined by the balance principle and A_n $0 \leq n \leq m$ are constants to be calculated.

4 Applications

In this section, we present the validity and useful of the four methods in solving the Wu–Zhang system [17]. Using the transformations $u(x,t)$, and $v(x,t)$ where $\xi = k(x - \lambda t)$, then system (1) is changed into

$$-\lambda\, u' + u\, u' + v' = 0 \tag{16}$$
$$-\lambda\, v' + (v\, u' + u\, v') + \frac{1}{3}k^2\, u''' = 0 \tag{17}$$

Equations (16) and (17) can be written respectively as:

$$-\lambda\, u' + \frac{1}{2}(u^2)' + v' = 0 \tag{18}$$
$$-\lambda\, v' + (u\, v)' + \frac{1}{3}k^2\, u''' = 0 \tag{19}$$

Integration equations (18) and (19) once with zero constant then

$$-\lambda\, u + \frac{1}{2}u^2 + v = 0 \tag{20}$$
$$-\lambda\, v + u\, v + \frac{1}{3}k^2\, u'' = 0 \tag{21}$$

From Eq. (20)

$$v = \left(\lambda - \frac{1}{2}\, u\right) u \tag{22}$$

Substitute (22) in (21) yields

$$\lambda^2 u - \frac{3}{2}\lambda\, u^2 + \frac{1}{2}u^3 - \frac{1}{3}k^2\, u'' = 0 \tag{23}$$

4.1. Csch function method

Now to apply Csch function substituting (10-11) into (23) yields:

$$A\, \mathrm{csch}^{\tau}(\mu\,\xi) - \frac{3}{2}\lambda\, A^2 \mathrm{csch}^{2\tau}(\mu\,\xi) + \frac{1}{2}A^3 \mathrm{csch}^{3\tau}(\mu\,\xi)$$
$$-\frac{1}{3}k^2\, A\, \tau\, \mu^2 \left((\tau + 1)\, \mathrm{csch}^{\tau+2}(\mu\xi) + \tau\, \mathrm{csch}^{\tau}(\mu\xi)\right) = 0 \tag{24}$$

From (24) balancing the power of Csch terms $3\tau = \tau + 2$, then $\tau = 1$

Substituting for $\tau = 1$ in (24) and Equating the exponents and the coefficients of each pair of the Csch functions, we find the following system of algebraic equations:

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$$\lambda^2 - \frac{1}{3}k^2\,\mu^2 = 0$$
$$\left(\frac{1}{2}A^2 - \frac{2}{3}k^2\,\mu^2\right) A = 0 \tag{25}$$

Solving the system in (25) to get:

$$\mu = \sqrt{3}\frac{\lambda}{k},\ A = 2\lambda \tag{26}$$

therefore

$$u(x,t) = 2\lambda\ \operatorname{csch}\left(\sqrt{3}\lambda\,(x-\lambda t)\right) \tag{27}$$

And

$$v(x,t) = \left(1 - \operatorname{csch}\left(\sqrt{3}\lambda\,(x-\lambda t)\right)\right) 2\lambda^2 \operatorname{csch}\left(\sqrt{3}\lambda\,(x-\lambda t)\right) \tag{28}$$

For $\lambda = 1$, then

$$u(x,t) = 2\operatorname{csch}\left(\sqrt{3}\,(x-t)\right) \tag{29}$$
$$v(x,t) = 2\left(1 - \operatorname{csch}\left(\sqrt{3}\,(x-t)\right)\right)\operatorname{csch}\left(\sqrt{3}\,(x-t)\right) \tag{30}$$

Figure (1) and (2) represent the solitary wave u(x,t), and v(x,t) in equation (29), (30) respectively.

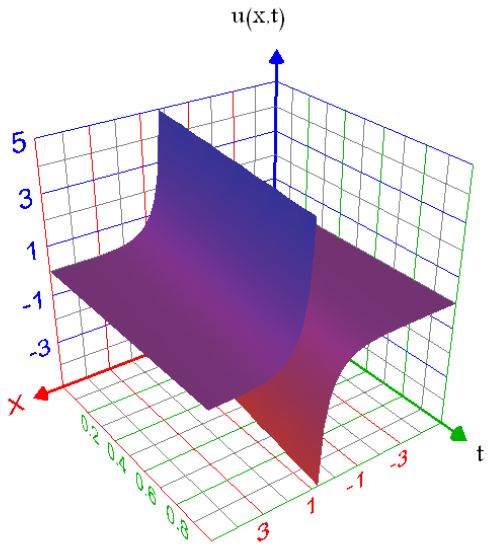

Figure 1. The solitary wave for u(x,t) in (29)

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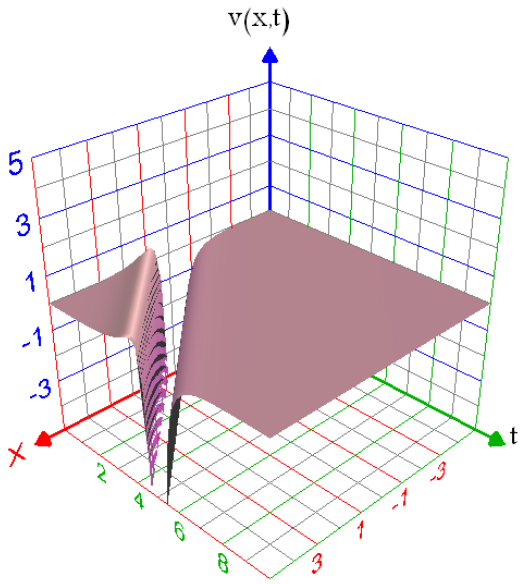

Figure 2. The solitary wave for v(x,t) in (30).

4.2. Tan-cot function method

Now to apply tan-cot function substituting (6-8) into (23) yields:

$$\lambda^2 A\tan^{\beta}(\mu\,\xi) - \frac{3}{2}\lambda\,A^2\tan^{2\beta}(\mu\,\xi) + \frac{1}{2}A^3\tan^{3\beta}(\mu\,\xi) - \frac{1}{3}k^2\,A\,\beta\mu^2\left[(\beta-1)\tan^{(\beta-2)}(\mu\xi) + 2\beta\tan^{\beta}(\mu\xi) + (\beta+1)\tan^{(\beta+2)}(\mu\xi)\right] = 0 \qquad (31)$$

From (31) we balance the power of tan in terms $3\beta = \beta + 2$, then $\beta = 1$

Substituting for $\beta = 1$ in (31) and equating the exponents and the coefficients of each pair of the tan functions, we find the following system of algebraic equations:

$$\lambda^2 - \frac{1}{3}k^2\,\mu^2 = 0$$
$$\left(\frac{1}{2}A^2 - \frac{2}{3}k^2\,\mu^2\right)A = 0 \qquad (32)$$

Solving the system in (32) to get:

$$\mu = \sqrt{3}\,\frac{\lambda}{k},\ A = 2\lambda \qquad (33)$$

therefore

$$u(x,t) = 2\lambda\,\tan\left(\sqrt{3}\lambda\,(x-\lambda t)\right) \qquad (34)$$

and

$$v(x,t) = \left(1 - \tan\left(\sqrt{3}\lambda\,(x-\lambda t)\right)\right)2\lambda^2\tan\left(\sqrt{3}\lambda\,(x-\lambda t)\right) \qquad (35)$$

For $\lambda = 1$, then

$$u(x,t) = 2\tan\left(\sqrt{3}\,(x-t)\right) \qquad (36)$$
$$v(x,t) = 2\left(1 - \tan\left(\sqrt{3}\,(x-t)\right)\right)\tan\left(\sqrt{3}\,(x-t)\right) \qquad (37)$$

Figure (3) and (4) represent the solitary wave u(x,t) and v(x,t) in equation (36), (37) respectively.

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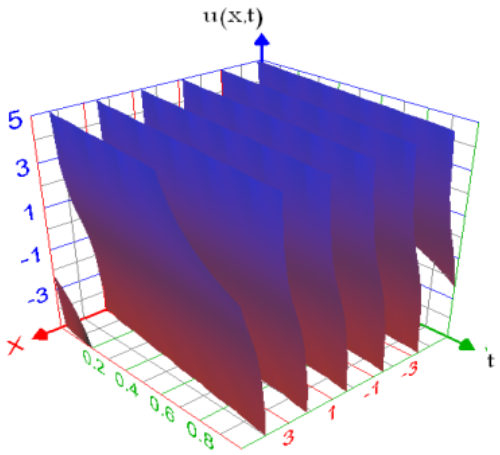

Figure 3. Represents u(x,t) in (36) for $-10 \le x \le 10$; $0 \le t \le 1$.

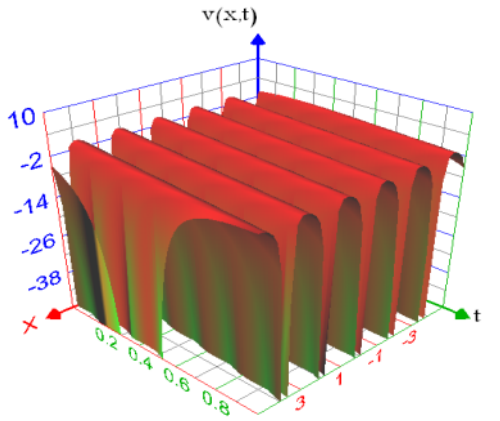

Figure 4. Represents v(x,t) in (37) for $-10 \le x \le 10$; $0 \le t \le 1$.

4.3. The Extended Tanh-Coth Method

By applying Tanh-Coth function method in (12-14) in eq.(23), and balance the highest-order linear term with the nonlinear terms. We balance u^3 with u'', to obtain: $3\,m = m + 2$, then $m = 1$.

The Extended Tanh-Coth method admits the finite expansion:

$$u = a_0 + a_1 Y + b_1 Y^{-1} \tag{38}$$

$$\frac{du}{dY} = a_1 - b_1 Y^{-2} \tag{39}$$

$$\frac{d^2u}{dY^2} = 2b_1 Y^{-3} \tag{40}$$

$a_0, a_1,$ and b_1 are constants to be determined. We Substitute Eqs.(38-40) into Eq.(23) to get

$$\lambda^2(a_0 + a_1 Y + b_1 Y^{-1}) - \frac{3}{2}\lambda\left(a_0{}^2 + 2a_1 b_1 + 2a_0 a_1 Y + a_1{}^2 Y^2 + 2a_0 b_1 Y^{-1} + b_1{}^2 Y^{-2}\right) + \frac{1}{2}\left(a_0{}^3 + 3\,a_0{}^2 a_1 Y + 3a_0 a_1{}^2 Y^2 + a_1{}^3 Y^3 + 3\,(a_0{}^2 Y^{-1} + 2a_0 a_1 + a_1{}^2 Y)\,b_1 + 3(a_0 Y^{-2} + a_1 Y^{-1})\,b_1{}^2 + b_1{}^3 Y^{-3}\right) + \frac{2}{3}k^2\left((a_1 Y - a_1 Y^3 - b_1 Y^{-1} + b_1 Y) - b_1(Y^{-3} - 2\,Y^{-1} + Y)\right) = 0 \tag{41}$$

Making the expressions at Y^i, $(i = -3, -2, -1, 0, 1, 2, 3)$ equal to zero, we get the following system of algebraic equations.

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$$
\begin{gathered}
\left(3{b_1}^2-4k^2\right)b_1=0\\
(\lambda-a_0){b_1}^2=0\\
2\lambda^2a_0-3\lambda\left({a_0}^2+2a_1b_1\right)+\left({a_0}^3+6\,a_0\,a_1\,b_1\right)=0\\
(6\lambda^2-18\lambda\,a_0+9({a_0}^2+\,a_1\,b_1)+4\,k^2)a_1=0\\
(\lambda-a_0){a_1}^2=0\\
(3{a_1}^2-4k^2)\,a_1=0
\end{gathered}
\tag{42}
$$

Solving the system of equations (42) we get:

$\lambda=\frac{4\,k}{3}$, $a_0=\frac{4\,k}{3}$, $a_1=b_1=\frac{2\,k}{\sqrt{3}}$

Therefore

$$
u(x,t)=\frac{2k}{3}\left\{2+\frac{2k}{\sqrt{3}}\left[\tanh\left(kx-\frac{4\,k^2}{3}\,t\right)+\coth\left(kx-\frac{4\,k^2}{3}\,t\right)\right]\right\} \tag{43}
$$

and

$$
\begin{gathered}
v(x,t)=\left(\frac{4\,k}{3}-\frac{k}{3}\left\{2+\frac{2k}{\sqrt{3}}\left[\tanh\left(kx-\frac{4\,k^2}{3}\,t\right)+\coth\left(kx-\frac{4\,k^2}{3}\,t\right)\right]\right\}\right)\\
\frac{2k}{3}\left\{2+\frac{2k}{\sqrt{3}}\left[\tanh\left(kx-\frac{4\,k^2}{3}\,t\right)+\coth\left(kx-\frac{4\,k^2}{3}\,t\right)\right]\right\}
\end{gathered}
\tag{44}
$$

For $k=\frac{3}{2}$,

$$
u(x,t)=\left\{2+\sqrt{3}\left[\tanh\left(\frac{3}{2}x-3\,t\right)+\coth\left(\frac{3}{2}x-3\,t\right)\right]\right\} \tag{45}
$$

and

$$
v(x,t)=\left(2-\frac{1}{2}\left\{2+\sqrt{3}\left[\tanh\left(\frac{3}{2}x-3\,t\right)+\coth\left(\frac{3}{2}x-3\,t\right)\right]\right\}\right)\left\{2+\sqrt{3}\left[\tanh\left(\frac{3}{2}x-3\,t\right)+\coth\left(\frac{3}{2}x-3\,t\right)\right]\right\} \tag{46}
$$

Figure (5) and (6) represent the solitary wave u(x,t), and v(x,t) in equation (45), (46) respectively.

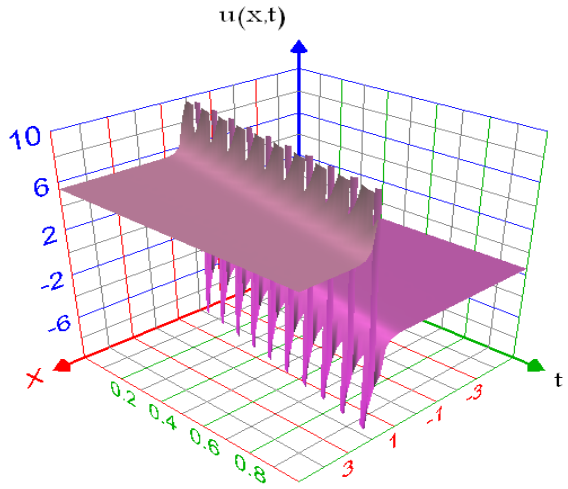

Figure 5. Represents u(x,t) in (45) for $-5\le x\le 5$; $0\le t\le 1$.

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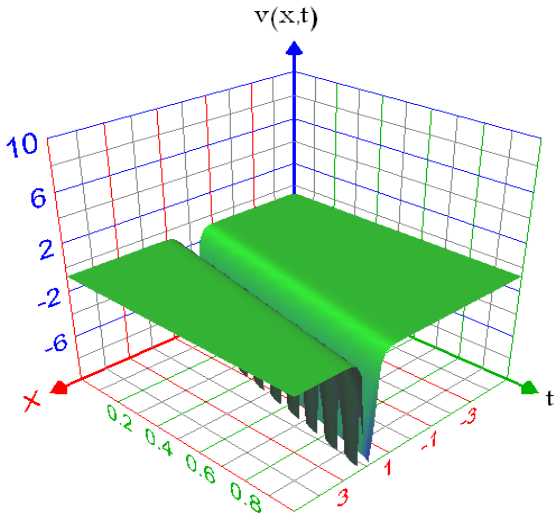

Figure 6. Illustrates v(x,t) in (46) for $-5 \le x \le 5$; $0 \le t \le 1$.

4.4 The Modified Simple Equation Method

Finally, let consider the function $u(\xi)$ in the following form.

$$u(\xi) = A_0 + A_1 \frac{\psi_\xi}{\psi} \tag{47}$$

where ψ is the unknown function and$\psi \neq 0$. Then, the first and second derivative of $u(\xi)$ are given by

$$u_\xi = A_1 \left(\frac{\psi\, \psi_{\xi\xi} - {\psi_\xi}^2}{\psi^2}\right)$$
$$u_{\xi\xi} = A_1 \left(\frac{\psi^2 \psi_{\xi\xi\xi} - 3\psi\, \psi_{\xi\xi}\psi_\xi + 2\, {\psi_\xi}^3}{\psi^3}\right) \tag{48}$$

substitute Eq.(47- 48) in (23) yield

$$\lambda^2 \left(A_0 + A_1 \frac{\psi_\xi}{\psi}\right) - \frac{3}{2}\lambda \left(A_0 + A_1 \frac{\psi_\xi}{\psi}\right)^2 + \frac{1}{2}\left(A_0 + A_1 \frac{\psi_\xi}{\psi}\right)^3 - \frac{1}{3}k^2 A_1 \left(\frac{\psi^2 \psi_{\xi\xi\xi} - 3\psi\, \psi_{\xi\xi}\psi_\xi + 2\, {\psi_\xi}^3}{\psi^3}\right) = 0 \tag{49}$$

Then Eq.(49) can be written as

$$\lambda^2 \left(A_0 + A_1 \frac{\psi_\xi}{\psi}\right) - \frac{3}{2}\lambda \left({A_0}^2 + 2A_0 A_1 \frac{\psi_\xi}{\psi} + {A_1}^2 \frac{{\psi_\xi}^2}{\psi^2}\right) + \frac{1}{2}\left({A_0}^3 + 3\, {A_0}^2 A_1 \frac{\psi_\xi}{\psi} + 3A_0\, {A_1}^2 \frac{{\psi_\xi}^2}{\psi^2} + {A_1}^3 \frac{{\psi_\xi}^3}{\psi^3}\right)$$
$$-\frac{1}{3}k^2 A_1 \left(\frac{\psi_{\xi\xi\xi}}{\psi} - 3\, \frac{\psi_{\xi\xi}\psi_\xi}{\psi^2} + 2\frac{{\psi_\xi}^3}{\psi^3}\right) = 0 \tag{50}$$

Equating expressions at ψ^3, ψ^2, ψ^1, and ψ^0 to zero, we have the following system of equations

$$\psi^3 : \left(3{A_1}^2 - 4k^2\right) A_1 {\psi'}^3 = 0 \tag{51}$$
$$\psi^2 : \{3[A_0 - \lambda]A_1 \psi' + 2\, k^2\, \psi''\} A_1 \psi' = 0 \tag{52}$$
$$\psi^1 : \{[6\lambda^2 - 18\lambda\, A_0 + 9{A_0}^2]\psi' - 2k^2\, \psi'''\} A_1 = 0 \tag{53}$$
$$\psi^0 : \left[6\,\lambda^2 - 9\lambda\, A_0 + 3{A_0}^2\right] A_0 = 0 \tag{54}$$

Now, solving Eq.(51)and Eq.(54), results in

$$A_0 = 0, \quad A_1 = \frac{2k}{\sqrt{3}} \tag{55}$$

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By substituting (51) into Eq.(52) and Eq.(53), and then solve the resulting system

$$\psi = c_1 + c_2\, e^{-\frac{\sqrt{3}\,\lambda}{k}\xi} \quad , \text{where} \quad \psi \neq 0 \tag{56}$$

$$\psi_\xi = -\,c_2 \frac{\sqrt{3}\,\lambda}{k}\, e^{-\frac{\sqrt{3}\,\lambda}{k}\xi} \tag{57}$$

where c_1, and c_2 are arbitrary constants.

Therefore:

$$u(x,t) = -2\,\lambda c_2 \frac{e^{-\sqrt{3}\,\lambda\,(x-\lambda t)}}{\left[c_1 + c_2\, e^{-\sqrt{3}\,\lambda\,(x-\lambda t)}\right]} \tag{58}$$

$$v(x,t) = -2\,\lambda^2 c_2 \frac{\left[c_1 + 2\,c_2\, e^{-\sqrt{3}\,\lambda\,(x-\lambda t)}\right] e^{-\sqrt{3}\,\lambda\,(x-\lambda t)}}{\left[c_1 + c_2\, e^{-\sqrt{3}\,\lambda\,(x-\lambda t)}\right]} \tag{59}$$

For $c_1 = c_2 = k = \lambda = 1$, then

$$u(x,t) = -2\,\frac{e^{-\sqrt{3}\,(x-t)}}{\left[1 + e^{-\sqrt{3}\,(x-t)}\right]} \tag{60}$$

$$v(x,t) = -2\,\frac{\left[1 + 2\,e^{-\sqrt{3}\,(x-t)}\right] e^{-\sqrt{3}\,(x-t)}}{\left[1 + e^{-\sqrt{3}\,(x-t)}\right]} \tag{61}$$

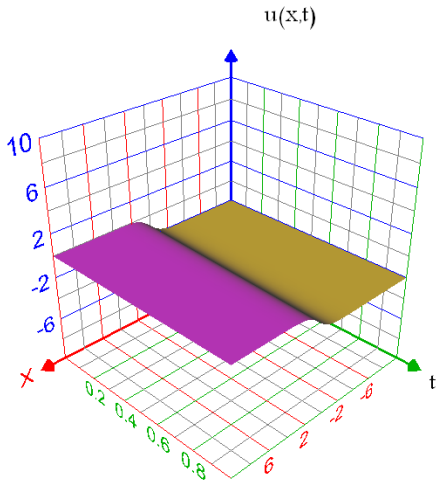

Figure 7. The solitary wave v(x, t) in (61) for $-10 \le x \le 10$; $0 \le t \le 1$.

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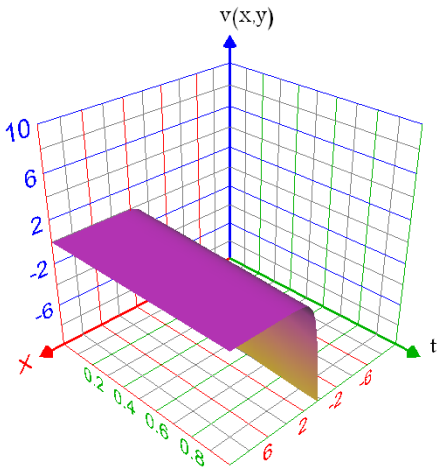

Figure 8. The solitary wave v(x, t) in (61) for $-10 \le x \le 10$; $0 \le t \le 1$.

5 Conclusion

Four methods the Csch method, Tan-Cot function method, Extended Tanh-Coth method, and MSEM are implemented to construct new solitary wave solutions for the Wu–Zhang system. Exact traveling wave solutions were obtained. Figures are illustrated and demonstrate the behaviour of extracted solutions that confirm the efficiency of the four methods in solving similar evolution equations in other problems.

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Conflict of Interests

There is no conflict of interests by authors regarding the publication of this manuscript.

References

[1] A. J. M. Jawad, M. Mirzazadeh, and A. Biswas, Solitary wave solutions to nonlinear evolution equations in mathematical physics, *Pramana,* **83**, 457-471 (2014).

[2] W. Malfliet, Solitary wave solutions of nonlinear wave equations, *Am. J. Phys*, **60**, 650-654 (1992).

[3] A. J. M. Jawad, Soliton solutions for the Boussinesq equations, *Journal of Mathematical and Computational Science*, **3,** 254 (2013).

[4] A. M. Wazwaz, Two reliable methods for solving variants of the KdV equation with compact and non-compact structures, *Chaos Solitons & Fractals*, **28**, 454-462 (2006).

[5] A. J. M. Jawad, M. Mirzazadeh, and A. Biswas, Dynamics of shallow water waves with Gardner-Kadomtsev-Petviashvili equation, *Discrete and Continuous Dynamical Systems, Series S.*, **8**, 1155-1164 (2015).

[6] E. Fan, Extended tanh-function method and its applications to nonlinear equations, *Phys Lett A*, **277**, 212-218 (2000).

[7] A. J. M. Jawad, M. Mirzazadeh, Q. Zhou and A. Biswas, Optical solitons with anti-cubic nonlinearity using three integration schemes, *Superlattices and Microstructures*, **105**, 1-10 (2017).

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[8] T. C. Xia, B. Li, and H. Q. Zhang, New explicit and exact solutions for the Nizhnik- NovikovVesselov equation, *Appl. Math. E-Notes*, **1**, 139-142 (2001).

[9] E. Yusufoglu, A. Bekir, Solitons and periodic solutions of coupled nonlinear evolution equations by using Sine-Cosine method, *Internat. J. Comput. Math*, **83**, 915-924 (2006).

[10] M. Inc and M. Ergut, Periodic wave solutions for the generalized shallow water wave equation by the improved Jacobi elliptic function method, *Appl. Math. E-Notes*, **5**, 89-96 (2005).

[11] Sh. Zhang, The periodic wave solutions for the (2+1)-dimensional Konopelchenko Dubrovsky equations, *Chaos Solitons & Fractals*, **30**, 1213-1220 (2006).

[12] Z. S. Feng, The first integer method to study the Burgers -Korteweg-de Vries equation, *J Phys. A. Math. Gen*, **35**, 343-349 (2002).

[13] A. J. M. Jawad, New exact solutions of nonlinear partial differential equations using Tan-Cot function method, *Studies in Mathematical sciences*, **5,** 13-25 (2012).

[14] A. R. Mitchell and D. F. Griffiths, *The Finite Difference Method in Partial Differential Equations*, John Wiley & Sons (1980).

[15] E. J. Parkes and B. R. Duffy, An automated tanh-function method for finding solitary wave solutions to nonlinear evolution equations, *Comput. Phys. Commun.* **98**, 288-300 (1998).

[16] A. J. M. Jawad, M. D. Petkovic and A. Biswas, Modified simple equation method for nonlinear evolution equations, *Appl Math Comput*., **217**, 869-77 (2010).

[17] M. Kaplan, P. Mayeli and K. Hosseini, Exact traveling wave solutions of the Wu–Zhang system describing (1 + 1)-dimensional dispersive long wave, *Opt. Quant. Electron*., **49**, 404 (2017).

[18] X. Zheng, Y. Chen and H. Zhang, Generalized extended tanh-function method and its application to (1+1)-dimensional dispersive long wave equation. *Phys. Lett. A*, **311**, 145–157 (2003).

[19] M. Mirzazadeh, M. Ekici, M. Eslami, E. V. Krishnan, S. Kumar and A. Biswas, Solitons and other solutions to Wu–Zhang system. *Nonlinear Anal. Modell. Control*, **22**, 441–458 (2017).

[20] M. Inc, B. Kilic, E. Karatas, M. M. Al Qurashi, D. Baleanu and F. Tchier, On soliton solutions of the Wu–Zhang system, *Open Phys,*. **14**, 76–80 (2016).

[21] A. J. M. Jawad, Three Different Methods for New Soliton Solutions of the Generalized NLS Equation, *Abstract and Applied Analysis*, 2017, Article ID 5137946 (2017).

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